Disputation: Adaptive Algorithms for Finite Element Methods Approximating Flow Problems

Klas Samuelsson, NADA, KTH, disputerar under ovanstående rubrik fredagen den 26 april kl. 14.15 i Kollegiesalen, administrationshuset (Valhallavägen 79).

Abstract

The thesis treats adaptive algorithms for solving partial differential equations, in particular flow problems. An adaptive computation has two purposes: firstly, to make an efficient use of the degrees of freedom by means of localized resolution, and secondly, to estimate the computational error. An automatic adaptive algorithm for the finite element method consists of the following ingredients: (1) A robust mesh generator, (2) a stable and accurate discretization method, (3) a stable and efficient solver of the system of equations, and (4) a reliable adaptive refinement criterion. The five papers in this thesis describe some new aspects of the ingredients (1)--(4).

The first paper considers pipe flow of generalized Newtonian fluids modeled by a quasi-linear elliptic boundary value problem of p-Poisson type. The equation is discretized by an adaptive finite element method and solved by the augmented Lagrangian method. A new scheme for adjusting the value of the crucial penalization parameter of the augmented Lagrangian is proposed.

The second paper considers a ring capacity problem with applications in the theory of quasiconformal mappings. An a posteriori estimate for the Grötzsch ring capacity gamma_n(s), dimension n >= 3, is proved and implemented in the adaptive finite element package PLTMG by Randolph E. Bank. The non-linear equation for the capacity problem takes a form treated in the first paper. The computational results are used to discuss several open problems involving gamma_3(s) and its inverse that occur in the theory of quasiconformal maps in 3-space.

In the third paper, an adaptive finite element method is compared to an integral interface equation method, developed by Johan Helsing, in order to compute the effective conductivity for anisotropic inclusions in anisotropic media. For the finite element computation, a posteriori bounds are presented for the effective conductivity. The finite element computations are performed by PLTMG.

In the fourth paper, multigrid methods for convective problems are analyzed. The analysis proves bounds on the residual damping of one V-cycle with pre- and post-smoothing. This damping depends on the choice of artificial diffusion, the number of smoothing steps and the approximated equation. The convergence rate results are compared with numerical experiments.

Finally, in the last paper, refinement algorithms for anisotropic meshes are developed. The anisotropic mesh refinement methods are based on recursively adding nodes at midpoints of triangle edges by estimating the interpolation error of the approximate solution. Approximation properties of the anisotropic refinements are compared to corresponding isotropic refinement methods.