Disputation: A Study of the Influence of Artificial Viscosity Terms on Solutions of Conservation Laws

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Abstract

In this thesis we study numerical solutions of conservation laws, obtained by using shock capturing, finite difference schemes.

For stationary shocks we find by linear analysis that in one dimension the smallest amount of viscosity needed to avoid oscillations is proportional to the characteristic speeds and the grid size. However, for the Euler equations it is found that the use of a scalar artificial viscosity term proportional to the smallest eigenvalue only yield very small oscillations in the case of weak and strong shocks. In two dimensions the influence of artificial viscosity terms is modeled by a scalar, discrete equation on a two-dimensionalgrid. Based on the solution of the model equation, two-dimensional artificial viscosity terms are found empirically.

We prove in one dimension that the analysis for stationary shocks is also valid for slowly moving shocks. A way to achieve well-behaved solutions for fast moving shocks is to calculate the shock on a grid that moves with the shock. The above-mentioned result implies that the speed of the grid needs not to be exactly equal to the speed of the shock for the steady state theory to be valid.

To analyze the solution of a conservation law containing contact discontinuities, we use as a model problem the first-order wave equation in one and two dimensions together with discontinuous initial data. When a second order method is used, oscillations appear due to phase speed errors in the numerical solution. We derive second order accurate schemes in which the phase speed errors are minimized in the lower frequencies and the higher frequencies are damped by an artificial viscosity term. Finally, we discuss the observation from calculations that if a first order method is used locally in the neighborhood of a shock, first order errors pollute the numerical solution downstream of a shock, although a higher order method is used in the downstream region. This only occurs for systems of equations with non-constant solutions. This effect is seen in a matched asymptotic analysis.

Keywords: artificial viscosity, central difference schemes, shock capturing, shocks, contact discontinuities, phase speed error.